The Influence of the Poloidal Magnetic Field on Mode Conversion of the Fast Magnetoronic Wave in a Two Component Toroidal Plasma

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ABSTRACT

Mode conversion from the fast magnetosonic wave to a slow wave can amount for the greatest part of power absorption in a two-ion-component toroidal plasma. It is shown that, with allowance for the correct form of the rotational transform, mode conversion in a cold plasma is only possible close to magnetic surfaces which do not cross the resonance i.e. the surface where the wave frequency is equal to the minority ion cyclotron frequency. Mode conversion at a layer which crosses the magnetic surfaces is possible if the plasma is hot and/or the layer is close to the resonance. In this case, however, mode conversion could be overshadowed by minority ion cyclotron damping, i.e. the perpendicular component of the wave vector could fail to reach values sufficiently large to allow electron Landau damping or second-harmonic majority ion cyclotron damping to contribute appreciably to the R.F. power absorption.

INTRODUCTION

Recent experiments at high power levels [1], [2] have shown that R.F. heating at the second harmonic of the ion cyclotron frequency is a very powerfull method of heating the plasma in a toroidal device. They have also provided striking evidence of the fundamental role that a minority of ions can assume in determining how and where RF power is deposited in the plasma if their cycoltron frequency is close to the wave frequency. If the concentration of the impurity ions is sufficiently large an electromagnetic wave can in fact, couple to an electrostatic one, which, in turn, will be damped either by electron Landau damping or by ion cyclotron damping. The coupling efficiency at the mode conversion layer has been studied in detail by treating the problem as one-dimensional, either by neglecting the poloidal magnetic field [3] or by assuming its normal component to be constant on a mode conversion layer [4]. As a consequence of these assumptions the mode conversion layer would cross the magnetic surfaces. It has already been pointed out [5], however, that only the component of the wave vector normal to a magnetic surface can become large. This is a direct consequence of the fact that the parallel component $\kappa_{\parallel}$ of the wave vector has an upper limit beyond which an electromagnetic wave cannot propagate, and that Snell's law requires that a wave propagate with a fixed κ_{φ}, the toroidal component of the wave vector. In a cold, uniform plasma one does in fact have $\kappa_{\parallel}^2 \leqslant |\sigma^{\pm}|$,where $\sigma^{\pm} = \mp \sum_j (\omega_{pj}/c^2)\,\omega^2/[\Omega_j(\omega \mp \Omega_j)]$; ω_{pj}, Ω_j being the ion plasma and the ion cyclotron frequency respectively, the sum is taken over ion species and we have assumed $|\omega - \Omega_j| \gg \kappa_{\parallel} u_j$, u_j being the ion thermal velocity. On the other hand, when finite ion temperature is taken into account, the field derivatives normal to a mode conversion layer will remain finite and mode conversion to a kinetic wave might in principle be possible at a layer which crosses the magnetic surfaces. In this case, however, the component of the field derivative parallel to the static magnetic field would become large,too so that mode conversion could be overshadowed by ion cyclotron damping. Owing to the great importance that the existence and shape of mode conversion layers can assume in future heating experiments in toroidal devices , it seems worthwhile to consider the problem in some more detail. First we evaluate an expression for the current density induced in a toroidal plasma by a low-amplitude electromagnetic wave whose frequency is close to the ion cyclotron frequency or its first harmonic. We then insert the value of the current density in Maxwell equations and look for the existence of mode conversion layers. First the plasma is assumed to be sufficiently cold so that a mode conversion layer, if present, has to coincide with a magnetic surface. We then discuss how the results of the cold plasma approximation must be modified when finite ion temperature is taken into account.

PLASMA CURRENT

An expression for the current density induced in a toroidal plasma by a low-amplitude electromagnetic wave has been evaluated in ref. [6] but just for the case where the frequency of the wave is equal to the ion cyclotron frequency on the axis and collisions are rare. We assume here that the collision frequency is sufficiently high $\nu \gg u_j/qR$ and evaluate a more general expression valid also for $\omega \neq \Omega_j$. We approximate a toroidal plasma by a straight plasma cyclinder, require that all quantities be periodic in z with period $2\pi R$, R being the major radius of the torus, and assume the static magnetic field bo be of the form

(1) $$\underline{B} = \underline{\theta}\, B_\theta + z\, B_z \;, \quad B_z = B_0\,(1 - r\cos\theta/R) \;, \quad B_\theta = r B_0/qR$$

We introduce rotating coordinates $V^{\pm} = V_r \pm V_\xi$ for the perpendicular components of the velocity $\underline{V}$, where $V_\xi = (V_\theta B_z - V_z B_\theta)/|B|$ and neglect terms of the order $\rho_j\, d(B_\theta/B_z)/dr$ (ρ_j being the Larmor radius), so that the motion of a charged particle in the magnetic field given by eq (1) reduces to

(2) $$\dot{V}^{\pm} = \mp \Omega_j V^{\pm} , \qquad u = \text{cost}$$

where $\Omega_j = \Omega_{jo}(1 - r\cos\theta/R)$, $\Omega_{jo} = e_j B_o/m_j c$ and $u = (V_z B_z + V_\theta B_\theta)/|B|$ is the component of $\underline{v}$ parallel to the static magnetic field. Equations (2) are, of course, incorrect for particles which are trapped or quasi trapped. It is assumed, however, that $r/R \ll 1$, so that their number will be small and their contribution to the plasma currents can be neglected. The value of the magnetic field encountered by a particle at a point (r,θ) is now approximated by the value at the gyrocentre $(\bar{r},\bar{\theta})$, whose equations of motion are assumed to be

(3) $$\bar{r} = r_o , \qquad \bar{\theta} = \theta_o + u(t' - t)/qR$$

We then obtain from eq (2)

$$V^{\pm} = V_o^{\pm} \exp[-i\varphi(t - t_o)]$$

(4) $$r = r_o + \Delta r , \qquad \theta = \theta_o + u(t - t_o) + \Delta\theta$$

where:

$$\Delta r = i\{V_o^+[e^{-i\varphi} - 1] - V_o^-[e^{+i\varphi} - 1]\}/(2\Omega_{jo})$$

$$\Delta\theta = \{V_o^+[e^{-i\varphi} - 1] + V_o^-[e^{+i\varphi} - 1]\}/(2\Omega_{jo})$$

$$\varphi = \Omega_{jo}(t - t_o) - (qr\Omega_{jo}/u)\{\sin[\theta_o + u(t - t_o)/qR] - \sin\theta_o\}$$

We now require the zero-order distribution function to be Maxwellian, $f_{oj} = [n_j/(\pi^{3/2} V_j^2 u_j)] \cdot \exp[-(V_r^2 + V_\xi^2)/V_j^2 - u^2/u_j^2]$

and derive an expression for the component of the current density perpendicular to the static magnetic field:

(6) $$J^{\pm} = \sum_j (e_j/m_j) \iiint_{-\infty}^{\infty} v^{\pm} f_{oj} \int_{-\infty}^{\infty} e^{-\nu(t'-t)} (v^{+\prime}E^{-\prime} + v^{-\prime}E^{+\prime})\, dt'$$

where n_j, m_j, e_j, V_j, u_j are the density, the mass, the charge and the perpendicular and parallel thermal velocities of the j species, respectively, $E^{\pm} = E_r \pm E_\vartheta$ are the rotating components of the electric field, ν is the collision frequency, and the primes indicate that a quantity is taken along the characteristics. Since the static magnetic field does not depend on z, we require the electric field of the wave to vary as

(7) $$E(r,\theta,z,t) = \sum_{m=-\infty}^{\infty} E_m(r) \exp[i(Nz/R + m\theta - \omega t)]$$

We further assume that the electric field does not change appreciably over a distance comparable with a Larmor radius, $E^{\pm}(r')\exp(im\theta') \approx [1 + im(\Delta\theta) - 1/2\, m^2(\Delta\theta)^2]\cdot[1 + (\Delta r)\partial/\partial r + 1/2\,(\Delta r)^2 \partial^2/\partial r^2]\cdot E_m(r)\exp\{im[\theta + u(t'-t)/qR]\}$, and perform on the right-hand side of eq (6) the integration over the perpendicular velocity first. Neglecting terms of the order r/R or $|Nq+m|u_j/(qr\Omega_{jo})$ with respect to unity and retaining terms of higher order in the ion Larmor radius only if they are proportional to $\exp(2i\varphi)$ we obtain

$$J^{-} \approx (\sigma^{-} c^2/4\pi i\omega)\, E^{-}$$

(8) $$J^{+} \approx \sum_j (\omega_{pj}/4\pi i\omega) \sum_{m=-\infty}^{\infty} \Big\{ -\omega/\Omega_{jo} + i\omega(u_j\sqrt{\pi})^{-1} \int_{-\infty}^{\infty} e^{-u^2/u_j^2} du \cdot \int_{-\infty}^{0} \exp\{[\nu - i\omega + i(Nq+m)u/qR]\tau\}\cdot[e^{i\varphi(\tau)} + 1/2\, \rho_j^2\, e^{2i\varphi(\tau)} \cdot (m^2/r^2 - \partial^2/\partial r^2)]\, d\tau \Big\} E^{+}(r)\exp[i(Nz/R + m\theta - \omega t)]$$

where one has $\sigma^- = \sum_j (\omega_{pj}^2/c^2)\,\omega^2/[\Omega_{jo}(\omega+\Omega_{jo})]$ and the sum is over ion species only since we have assumed charge neutrality to write $\omega_{pe}^2/\Omega_e = \sum_j \omega_{pj}^2/\Omega_j$. We now expand exp $[is\varphi(t)]$, s = 1,2 in a series of Bessel functions, so that J^+ can be written in the form

(9) $$J^+ \approx \sum_j (\omega_{pj}^2/4\pi i\omega) \sum_{m=-\infty}^{\infty} \exp[i(Nz/R + m\theta - \omega t)] \cdot \sum_{n=-\infty}^{\infty} [\delta_{m,n} + T_{m,n}^{(1)} + 1/2\, \rho_D^2\, T_{m,n}^{(2)} (n^2/r^2 - \partial^2/\partial r^2)] E_n^+$$

where:

(10) $$T_{m,n}^{(s)} = i(\omega/u_j\sqrt{\pi})\, e^{i(m-n)\pi/2} \int_{-\infty}^{\infty} e^{-u^2/u_j^2} du \int_0^{\infty} J_{(m-n)}[(2s\,\Omega_{jo} q r/u_j) \sin(u\tau/2qr)] \exp\{-\nu\tau - i[s\Omega_{jo} - \omega + (2N+m+n)u\tau/qR]\} d\tau$$

and $\delta_{m,n}$ is the Kroencker symbol. We now assume collisions to be sufficiently large $\nu \gg u_j/qR$, so that in the limit $u_j/\Omega_j \to 0$ the main contribution of the integral over τ comes from the neighbourhood of $\tau = 0$. We then substitute in the argument of the Bessel function $\sin(u\tau/2qr) \approx u\tau/2qR$ and, performing the integration over τ, we obtain

(11) $$T_{m,n}^{(s)} \approx [\omega R/(s\Omega_{jo} r|\epsilon|\sqrt{\pi})] \exp[i|m-n|\pi/2] \cdot \int_{\infty - i\pi/2}^{\infty + i\pi/2} \exp[(\sinh z - \gamma)^2/\epsilon^2 - |m-n| z]\, dz$$

where

$$\epsilon = [2Nq + m + n]\, u_j/(2sq\Omega_{oj} r)$$

$$\gamma = \nu R/(s\Omega_{oj} r) + i\Delta/r$$

and $\Delta = -(\omega - s\Omega_{oj})R/(s\Omega_{oj})$ is the distance of the plane $\omega = s\Omega_j$ from the axis of the plasma cylinder. If one has $\Delta = 0$

and collisions are neglected, the integral on the right-hand side of eq (11) can be taken exactly so that

(13) $$T^{(s)}_{m,n} = i\,[\omega R/(s\Omega_{jo}\gamma|\epsilon|)]\cdot I_{(m-n)/2}(1/2\epsilon^2)\exp[i|m-n|\pi/2 - 1/2\epsilon^2]$$

while if $\Delta \neq 0$ only an approximate solution can be found. In the limit $\varrho_j \to 0$ we then have:

(14) $$T^{(s)}_{m,n} \approx [\omega R/(s\Omega_{jo}\gamma)]\cdot[\sqrt{2\pi}/(2\pi|\gamma\epsilon|)]^{1/2}\cdot[(i+\Delta/|\Delta|)$$
$$K_{1/4}(\mu) + i\pi\sqrt{2}\,(|\sqrt{1+\gamma^2}|/\sqrt{1+\gamma^2})\,I_{1/4}(\mu)]\cdot$$
$$\exp[-\mu + \gamma^2\delta^2/\epsilon^2 - |m-n|(z_0 - i\pi/2)]$$

where

(15) $$\mu = 1/2\,|(1+\gamma^2)/(\gamma\epsilon)|^2$$

$$\sinh z_0 = \gamma\cdot(1+\delta) \qquad Re\{z_0\} > 0$$

$$\delta = [(1+\gamma^2)/6\gamma^2]\cdot[(a^+ + a^-) - i\sqrt{3}\,(a^+ - a^-) - 1]$$

$$a^\pm = 1/2\,[\beta \pm i\sqrt{1-\beta^2}\,]^{1/3}, \quad \beta = 1 - (|m-n|\epsilon^2/4\gamma^2)^2\cdot[6\gamma^2/(1+\gamma^2)]^3$$

Here K_ν, I_ν indicate Bessel functions in the notation of G.N. Watson [7]. If γ is not too close to Δ so that $\mu \gg 1$ and $|\delta| \ll 1$ in eqs (15), then eq (14) can be written in a much simpler form

(16) $$T^{(s)}_{m,n} \approx i\,[\omega R/(s\Omega_{jo}\gamma\sqrt{1+\gamma^2})]\cdot[\sqrt{1+\gamma^2}+\gamma\,]^{-|m-n|}$$
$$\exp[\gamma^2\delta^2/\epsilon^2 - |m-n|\cdot(\gamma\delta/\sqrt{1+\gamma^2} - i\pi/2)]$$

and $\delta \approx 1/2\,|(m-n)\epsilon^2|/(\gamma\sqrt{1+\gamma^2})$, $\quad Re\{\sqrt{1+\gamma^2}\} > 0$

In the most interesting case $r > \Delta$, i.e. at magnetic surfaces which cross the plane $\omega = \Omega_j$, we then have

(17) $$J^+ = (\omega_{pj}^2/4\pi\Omega_{oj})\cdot\{-E^+(r,\theta) + i R/(\sqrt{r^2+\Delta^2})\cdot\sum_{m=-\infty}^{\infty} E_m^+(r)$$
$$\exp[i(Nz/R + m\theta - \omega t)]\cdot\sum_{n=-\infty}^{\infty}[\sqrt{1-(\Delta/r)^2} + i(\Delta/r)]$$
$$\exp[i(n\cdot\theta + |n|\pi/2 - (n/2)^2[Nq + m + n/2]^2\cdot[u_j/(q\Omega_{jo}\sqrt{r^2-\Delta^2})]^2\}$$

Here it holds that $\Delta = -(\omega - \Omega_{oj})R/\Omega_{oj}$ and for simplicity we have assumed that there is only one ion species and that $\omega \approx \Omega_{jo}$. From eq (16) it then follows that the current density remains finite at the resonance $\omega = \Omega_j$ even if the electric field does not change along the line of force of the static magnetic field:

(18) $$|J^+| \leq (\omega_{pj}^2/4\pi\omega)\cdot(R/\sqrt{r^2-\Delta^2})\cdot(q\Omega_{jo}\sqrt{r^2-\Delta_j}/u_j)^{1/2}$$
$$\sum_{m=-\infty}^{\infty} |E_m^+|\cdot[1 + |qN+m|\cdot u_j/(q\Omega_{oj}\sqrt{r^2-\Delta^2})]^{-1}$$

when $q\to\infty$ eq (18) yields the well-known relation $|J^+| \lesssim (\omega_{pj}^2/4\pi\omega)$ $|\omega/K_{\parallel}u_j|\cdot|E^+|$ while if $|Nq + n| \ll (q\Omega_{jo}\sqrt{r^2-\Delta^2}/u_j)$ the current density is partically independent of $K_{\parallel}$. This is obviously due to the fact that owing to the rotational transform the particle passes through the resonance and cannot remain in phase with the electric field of the wave. The cold-plasma approximation for the current density $J^+ = \{(\omega_{pj}^2/4\pi\omega)\omega^2/[\Omega_j(\omega - \Omega_j)]\}E^+$ can easily be obtained form eqs (9), (15) if it is assumed that:

(19) $$|(\Delta - r\cos\theta)E^+| \ll (u_j q\Omega_{jo} r)^{1/2}|1 + (u_j/q\Omega_{oj} r)^{1/2}\cdot(Nq + \partial/\partial\theta)E^+|$$

DISPERSION RELATION

We neglect in the following electron inertia and terms proportinal to r/qR with respect to unity, so that we get $E_{\parallel} \approx 0$ and can set in Maxwell equations $E_\theta \approx E_\zeta$, $E_z \approx -(r/qR)E_\zeta$, $J_\theta \approx J_\zeta + (r/qR)J_{\parallel}$ and $J_z \approx J_{\parallel} - (r/qR)J_\zeta$. We again assume the electric field to vary as $\underline{E}(r,\theta,z,t) = \sum_{m=-\infty}^{\infty} E_m(r) \exp[i(Nz/R + m\theta - \omega t)]$ and introduce rotating coordinates for the perpendicular components of the current density. Setting $(4\pi i\omega/c^2)J^- = \sigma^-(E_r - iE_\zeta)$ and substituting $E_r = E^+ + iE_\zeta$ we then obtain from Maxwell equations $(E_\theta \approx E_\zeta = E)$:

$$(h/r)E_m' - [C_m + (h-1)h/r^2]E_m = -i[\Psi_m - (h^2/r^2)E_m^+]$$

(20) and

$$(h/r)\Psi_m' + [C_m - h'/r + h(h+1)/r^2]\Psi_m = -iB_m E_m$$

where

$$\Psi_m = [C_m + \sigma^-/2]E_m^+ - (2\pi i\omega/c^2)J_m^+$$

$$C_m = (N/R + m/qR) - \sigma^-$$

(21) $$B_m = C_m(C_m - h'/r) + hC'/r$$

$$h = m - Nr^2/qR^2$$

and the primes indicate derivatives with respect to r. Eliminating E from eqs (21), we finally obtain

(22) $$\Psi_m'' + [1/r - B_m'/B_m]\Psi_m' - [(h+1)^2 + 3h'/r + 2C_m + H_m/B_m]\Psi_m = -B_m E_m^+$$

and $H_m = (h+1)B_m'/r + 2(C_m')^2 + r(C_m - h'/r)(C_m'/r)'$

We now consider an electromagnetic wave propagating in a two-ion-component plasma with $\omega \approx \Omega_H > \Omega_D$ and $n_H/n_D \ll 1$. From eqs (9), (21) we then have

(23) $$\Psi_m = \sum_{n=-\infty}^{\infty} d_{m,n} E_n$$

where

$$d_{m,n} = A_m \delta_{m,n} - 1/2\,(\omega_{PH}^2/c^2)\,T_{m,n}^{(1)} - 1/4\,(\omega_{PD}^2/c^2)$$

(24) $$T_{m,n}^{(2)}\,\rho_D^2\,(n^2/r^2 - \partial^2/\partial r^2)$$

$$A_m = (N/R + m/qR)^2 + \omega_{PD}^2\,\omega^2/[c^2(\omega^2 - \Omega_{DO}^2)]$$

and $\delta_{m,n}$ is the Kroenecker symbol. Since we are considering an electromagnetic wave, terms proportinal to ρ_D^2 will, in general, be small and could be neglect in eqs (24). However, we are concerned here with the problem of finding mode conversion layers, i.e. just those surface in the neighbourhood of which the field derivatives of an electromagnetic wave become so large that these terms become comparable with the other terms of eq (24). We first simplify the problem by assuming the plasma to be cold. Then since it holds that $\lim_{\rho_D \to 0} \{\rho_D^2 \sum_{n=-\infty}^{\infty} | n^2 T_{m,n}^{(2)}/A_n |$ $= 0$ as can be seen from eqs (13), (14) once the ratio u_D/v_D is assumed to be finite, it is clear that only terms proportional to $\rho_D^2\,\partial^2 E^+/\partial r^2$ can become large in eqs (24) and that a mode conversion layer, if present, must coincide with a magnetic surface $r = \bar{r}$. Away from a mode conversion layer an electromagnetic wave will propagate with finite values of $\partial^2 E^+/\partial r^2$, so that in the limit $v_j \to 0$ $u_j \to 0$ we have:

(25) $$d_{m,n} \approx A_m \delta_{m,n} - i/2\,[\omega_{PH}^2\,\omega R/(c^2 \Omega_{HO}\, r\sqrt{1+\gamma^2})]\,(-i\alpha)^{-|m-n|}$$

where $\gamma = \nu R/\Omega_{HO} r + i\,\Delta_H/r$, $\Delta_H = -(\omega - \Omega_{HO})R/\Omega_{HO}$
$\alpha = \sqrt{1+\gamma^2} + \gamma$, and the principal value of $\sqrt{1+\gamma^2}$ must be taken if $|\gamma| > 1$.
The solution of eq. (23) will then be of the form $E_m^+ = \sum_{n=-\infty}^{\infty} D^{-1}_{m,n} \Psi_n / D$ where $D^{-1}_{m,n}$ is the cofactor of $d_{m,n}$, and D the value of the infinite determinant $|d_{m,n}|$. It is then clear that $\partial^2 E^+/\partial r^2$ will become large if it is possible to find a value $\bar{r}$ of r such that $D(\bar{r})$ and r is sufficiently close to $\bar{r}$. The problem of finding mode conversion layers is therefore reduced to finding the roots of $D(r) = 0$. It is now easy to show (see Appendix) that no solutions of $D(r)=0$ exist if $r > |\Delta|$, i.e. no mode conversion to a kinetic wave takes place in a toroidal plasma at magnetic surfaces which cross the plane $\omega = \Omega_H$. Similarly it is found that no solutions of $D(r) = 0$ can be obtained if $\Delta_H < 0$ i.e. $\omega > \Omega_{HO}$ or if the concentration of the minority ions $\eta = \omega_{PH}/\omega_{PD}$ is too low, and that

(26) $$\eta(\omega^2 - \Omega_{DO}^2)R/\omega\Omega_{DO} > \Delta_H - \bar{r} > 0$$

is a necessary condition for the existence of mode conversion layer in a cold, toroidal plasma. On the other hand, it can be proved that there exist reasonable values of the plasma parameters satisfying eqs (26) for which $D(r) = 0$. Strictly speaking, what can be proved (see Appendix) is that for any given value $\bar{r} < \Delta_H$ of r there exists at least one value η^* of η ,

(27) $$(\Delta_H - r) < \eta^* R(\omega^2 - \Omega_{DO}^2)/\omega\Omega_{DO} < \Delta_H$$

such that $D(\bar{r}) = 0$.
These results become more intuitive if we transform back eq (23). We then obtain

(28) $$\Psi(r,\theta) = (K_{\parallel}^2 - \varepsilon_{\parallel}) \cdot E^+(r,\theta)$$

where $\varepsilon_{\parallel} = -(\omega_{PD}^2/c^2)\{\omega^2/(\omega^2 - \Omega_{DO}^2) + 1/2\,\eta\,\omega^2/[\Omega_{HO}(\omega - \Omega_H + i\nu)]$
and $K_{\parallel} = [N/R + (1/qR)\partial/\partial\theta]$ so that the roots of $D(r)$ correspond

to periodic solutions of the differential equation $(K_{\parallel}^2 - \varepsilon_{\parallel}) \cdot E^+ = 0$. In the limit $q \to \infty$ we then have the well-known relation $N^2/R^2 - \varepsilon_{\parallel} = 0$ and the mode conversion layers degenerate in the open layer of ref [3]. Obviously the infinite determinant $|d_{m,n}|$ does not converge in this case. No periodic solutions of $(K_{\parallel}^2 - \varepsilon_{\parallel}) E^+ = 0$ can be expected if $r > \Delta_{H0}$ since then the losses at $\omega = \Omega_H$ cannot be accounted for. If we now define a cut-off of an electromagnetic wave as the surface where $\partial\Psi/\partial r,\ \partial^2\Psi/\partial r^2 = 0$, then the cut-off surfaces can be found in a way analogous to the mode conversion layer. For eq (25) yields $\partial\Psi/\partial r,\ \partial^2\Psi/\partial r^2 = 0$ at $r = r^*$ if

(29) $$[(h+1)^2/r^2 + 2C_m + H_m/B_m]\Psi_m - B_m E_m^+ = 0$$

at $r = r^*$. Thus since eq. (29) admits non trival solutions only if $D^*(r^*) = 0$, where $D^*(r) = |d^*_{m,n}| = |A^*_m \delta_{m,n} + d_{m,n}|$

$$A^*_m = B_m / [(h+1)^2/r^2 + 2C + H_m/B_m]$$

the equation of a cut-off surface will be $r = r^*$ with $D(r^*) = 0$. Therefore the smaller the ratio $|A_m/d_{m,m}|$ the closer a cut-off surface will be to a mode conversion layer and hence the less effective mode conversion will be. Let us now give some order-of-magnitude estimate of how the results of the cold-plasma approximation must be modified when finite ion temperature is taken into account. For simplicity we set $N = 0$ and first assume $|\Delta_H - r\cos\theta| \gg (u_H/q\Omega_{H0} r)^{1/2} [1 + (u_H/q\Omega_{H0} r)^{1/2} [1/|E^+| \, |\partial^2 E^+/\partial\theta^2|]$, so that ion parallel motion can be neglected. Transforming back eq. (21), we then obtain

(30) $$\Psi(r,\theta) = \{ -(1/qR)^2 \partial^2/\partial\theta^2 + (\omega_{PD}^2/c^2)[\omega/(\omega^2 - \Omega_{D0}^2) - 1/2\,\eta\omega R/[\Omega_{D0}(\Delta_H - r\cos\theta) - 1/4\,[\rho_D^2 R/(\Delta_D - r\cos\theta) \cdot (1/r^2 \partial^2/\partial\theta^2 + \partial^2/\partial r^2)\} E^+(r,\theta)$$

where $\Delta_D = -(\omega - 2\Omega_{D0})R/(2\Omega_{D0})$. If it now holds that $|\Delta_D| \geq |\Delta_H| > r$ and $\Delta_H - r$ is sufficiently large, then eq (30) holds for all values of θ and, as long as $\omega_{PD}^2 \rho_D^2/c^2 \ll 4(r/qR)^2 \cdot R/(\Delta_D - r)$ the results of the cold-plasma approximation are little affected by finite ion temperature, except of course close to a surface

$r = \bar{r}$, $D(\bar{r}) = 0$ Note, however, that close to a mode conversion layer $r = \bar{r}$ an electromagnetic wave propagates with rather large values of $k_\parallel$. For close to $\bar{r}$ we have in fact $|(1/qR)^2 \partial^2 E^+/\partial\theta^2|$ $\lesssim 1/2\, \eta R/(\Delta_H - r)$ so that terms of the order $(\omega - \Omega_H)^2/(k_\parallel u_H)^2$ $\gtrsim [(r-\Delta_H)/R]^3 \Omega_{Ho} c^2/(\eta\, \omega_{PD}^2 u_H^2)$ can become comparable with or even less than unity if $\bar{r}$ is too close to Δ, in which case ion cyclotron damping can no longer be neglected and may possibly overshadow mode conversion. This is due to the fact that the closer an electromagnetic wave approaches the surface $r = \bar{r}$ the more and more its group velocity points in the θ direction. The wave then propagates around the surface $r = \bar{r}$ toward regions of decreasing magnetic field strengh, while the parallel component at the wave vector increases (magnetic beach effect [6]). Thus if $\bar{r}$ is too close to Δ_H, the wave will be absorbed by the plasma via ion cyclotron damping before undergoing mode conversion. In this way one could in fact explain the high direct ion heating observed in the T.F.R. tokamak [9]. On the other hand, if one has $r > |\Delta_H|$ and close to the surface $r\cos\theta = \bar{x}$, $\bar{x} = [\Delta_H - 1/2\, \eta R (\omega^2 - \Omega_{Do}^2)/\omega\, \Omega_{Ho}]$ it holds that $|\Delta_D - r\cos\theta| \ll 1/4\, R\, (\omega_{PD}^2 \rho_D^2/c^2)\cdot(qR/r)^2$ as could be the case in a deuterium plasma with a minority of hydrogen ions, $\Delta_D = \Delta_H$, then mode conversion to a kinetic wave can be effective close to the surface $r\cos\theta = \bar{x}$ and the influence of the rotational transform can be neglected. Here again mode conversion can be overshadowed by ion cyclotron damping. In fact the normal component of the field derivatives of an electromagnetic wave becomes large on approaching a mode conversion layer $|\partial^2 E^+/\partial x^2 \approx (\omega_{PD}/c\rho_D)(|\Delta_D - \bar{x}|/R)^{1/2}$ Then since the layer crosses the magnetic surfaces $k_\parallel$ will become large, too $k_\parallel^2 \approx 1/qR\, \partial^2 E^+/\partial\theta^2 \approx r^2/q^2R^2\, \partial^2 E^+/\partial x^2$, so that if $\bar{x}$ is too close to Δ_H i.e $[(\Delta_H - \bar{x})/R]^2 \lesssim (r/qR)^2 (\omega_{PD}\, \rho_H^2/\rho_D c)$ $(|\Delta_D - \bar{x}|/R)^{1/2}$ an electromagnetic wave will be strongly damped before reaching the mode conversion layer. Note that, in order for an electromagnetic wave to reach the surface $r\cos\theta = \bar{x}$ undamped and be mode-converted there, one requires that:

$$(qR/r)^2 (\rho_H/\rho_D)^2 [(\Delta_H - \bar{x})/R]^2 [R/|\Delta_D - \bar{x}|]^{1/2} \gg (\omega_{PD}\rho_D/c) \gg (r/qR)(|\Delta_D - \bar{x}|/R)^{1/2}$$

CONCLUSIONS

An expression for the current density induced by a low amplitude electromagnetic wave in a plasma confined by a toroidal magnetic field with finite values of the rotational transform has been evaluated. The wave frequency has been assumed close to the ion cyclotron frequency or its first harmonic. The condition necessary for mode conversion from an electromagnetic wave to an electrostatic one has been evaluated on the assumption that the plasma is cold. It is found that if the wave frequency is less that the ion cyclotron frequency of the minority ion component on the axis mode conversion is possible at magnetic surfaces which do not cross the plane where the wave frequency is equal to the local value of the ion cyclotron frequency. When finite temperature effects are taken into account it is found that mode conversion becomes possible at layers which do not coincide with a magnetic surface, but only if the plasma is sufficiently hot and the mode conversion layer is not too close to the plane $\omega = \Omega_H$

APPENDIX

We investigate in the following the existence of roots of the equation $D(r) = 0$, where $D(r)$ is the infinite determinant $|d_{m,n}|$ and the quantities $d_{m,n}$ are given in eqs (24), (25). To this purpose let us first write $D(r)$ in a more usable form:

A (1) $$D(r) = \lim_{M \to \infty} \left(D_M \prod_{-M}^{M} A_m \right)$$

where $D_M = | d_{m,n} / A_m |$ $\quad -M \leq (m,n) \leq M$

Since the quantities A_m are positive and D_M is, for finite values of q , absolutely convergent, the roots of $D(r)$ will be close to the roots of $D_{\bar{M}}$ if $\bar{M}$ is chosen sufficiently large. After some simple algebra $D_{\bar{M}}$ can be reduced to the form

A (2) $$D_{\bar{M}} = (i\alpha)^{2M+1} \, |g_{m,n}| / (1+\alpha^2)$$

where

$$g_{m,n} = \alpha_m \delta_{m,n} + \delta_{|m|,|m|+1} + \delta_{|m|,|m|-1}$$

A (3) $$\alpha_{\pm M} = i/\alpha + \omega_{PH}^2 \omega R / (A \pm M \Omega_{HO} c^2 r)$$

$$\alpha_m = 2 [i\gamma + \omega_{PH}^2 \omega R / (A_m \Omega_{HO} c^2 r)$$

and $\alpha = \sqrt{1+\gamma^2} + \gamma$, $\gamma = \nu R / \Omega_{HO} r + i\Delta / r$

$\Delta = -(\omega - \Omega_{HO}) R / \Omega_{HO}$ For simplicity we now assume N = 0 and expand $|g_{m,n}|$ according to the row so that $|g_{m,n}| = 2 G_M^{(0)} G_M^{(1)}$ and the quantities $G_M^{(0)}$, $G_M^{(1)}$ can be obtained from the recurrence relation

A (4) $$G_m = \alpha_m G_{m-1} - G_{m-2}$$

with $G_{-1}^{(0)} = 1$, $G_0^{(0)} = \alpha_0 / 2$ and $G_0^{(1)} = 1$, $G_1^{(1)} = \alpha_1$

It is now easy to show that no real solution of $G_{\bar{M}}^{(0)}(r) = 0$ (or $G_{\bar{M}}^{(1)}(r) = 0$) exist if $r > |\Delta|$. For, with $|\Delta/r| \leq 1$ one has neglecting collisions, $\gamma \doteq i\Delta/r$ and $\alpha = \sqrt{1-(\Delta/r)^2} + i(\Delta/r)$ so that $\alpha_{\bar{M}}$ is complex while the quantities α_m are real. It then follows from the recurrence relation that the quantities $G_m(r)$ are real for $m \neq \bar{M}$, so that in order to have $G_{\bar{M}}(r) = 0$ it should hold that $G_{\bar{M}-1}(r) = 0$ and $G_{\bar{M}-2}(r) = 0$. This is not possible however, since then it would follow that $G_m(r) = 0$ for all values of m, whereas we have $G_{-1}^{(0)} = 1$ and $G_0^{(1)}(r) = 1$.
Let us now assume that $|\Delta/r| > 1$ and prove that no real solutions of $G_{\bar{M}}(r) = 0$ exist if $\Delta < 0$ $(\omega > \Omega_{HO})$. For, with $\Delta < 0$ one has $\alpha_m > 2$ for all m, so that $G_m(r)/G_{m-1}(r) > 2 - G_{m-2}(r)/G_{m-1}(r)$ and since $G_0^{(0)}(r) / G_{-1}^{(0)}(r) = \alpha_0/2 > 1$ it then follows that $G_{\bar{M}}^{(0)}(r) = \alpha_{\bar{M}} G_{\bar{M}-1}^{(0)}(r) - G_{\bar{M}-2}^{(0)}(r) \neq 0$.
It is now easy to prove that $G_{\bar{M}}(r) \neq 0$ if $\Delta > 0$ and $\eta < \bar{\eta}$ $= \omega \Omega_{HO} |\Delta_H - r| / [(\omega^2 - \Omega_{DO}^2) R]$ since then it holds that $\alpha_m < \alpha_0 < -2$, so that $G_m^{(0)}(r) / G_{m-1}^{(0)}(r) < 1$ and $\alpha_{\bar{M}} G_{\bar{M}-1}^{(0)}(r) / G_{\bar{M}-2}(r) > 1$.

In a similar way it can be proved that $G^{(1)}_{\bar{M}}(r) \neq 0$ if $\Delta_H < 0$ or $\eta < \bar{\eta}$, so that $\eta > \bar{\eta} > 0$ is a necessary condition in order to have roots of $D(r) = 0$. We shall now prove that for any given value $\bar{r} < \Delta$ of r it is always possible to find a value $\eta^* > \bar{\eta}$ of η such that $G^{(0)}_{\bar{M}}(\bar{r}) = 0$. For, since $G^{(0)}_m(\bar{r})$ is a polynominal of degree m + 1 in η, there will be m + 1 values η_s $(s = 0, 1, \dots m)$ of η for which $G^{(0)}_m(\bar{r}) = 0$. Let us first prove that if the zeros of $G^{(0)}_m(\bar{r})$ are real and intersect the zeros of $G^{(0)}_{m-1}(\bar{r})$ i.e. $\eta^0_m < \eta^0_{m-1} < \dots < \eta^{m-1}_{m-1} < \eta^m_m$ then the zeros of $G^{(0)}_{m+1}(\bar{r})$ are real and intersect the zeros of $G^{(0)}_m(\bar{r})$. For, since $G^{(0)}_{m+1}(\bar{r}) = -G^{(0)}_{m-1}(\bar{r})$ when $\eta = \eta^s_m$ and we have assumed that $G^{(0)}_{m-1}(\bar{r})$ changes sign once for $\eta^{s-1}_m < \eta < \eta^s_m$ so will $G^{(0)}_{m+1}(\bar{r})$; on the other hand for $\eta \to \pm\infty$ $G^{(0)}_{m+1}(\bar{r})$ has the same sign as $G^{(0)}_{m-1}(\bar{r})$, and since we have assumed that $G^{(0)}_{m-1}(\bar{r})$ does not change sign for $\eta < \eta^0_{m-1}$ or $\eta > \eta^0_{m+1}$, there will be a value of $\eta < \eta^0_m$ and a value of $\eta > \eta^m_m$ for which $G^{(0)}_{m+1}(\bar{r}) = 0$ so that the zeros of $G^{(0)}_{m+1}(\bar{r})$ will be real and will intersect the zeros of $G^{(0)}_m(\bar{r})$. We now have $G^{(0)}_0(\bar{r}) = \alpha/2$ so that $\eta^{(0)}_0 = \omega\Omega_{HO}\Delta_H / [(\omega^2 - \Omega^2_{DO})R]$ and since, as is easy to see, $\eta^0_1 < \eta^0_0 < \eta^1_1$ for $r > \Delta$ it then follows that there will be at least one value η^* of η for which $G^{(0)}_{\bar{M}}(\bar{r}) = 0$ and $\bar{\eta} < \eta^* < \eta^0_0$.

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